

# Exotic pentaquarks as Gamov–Teller resonances

Dmitri Diakonov<sup>1,2</sup>

<sup>1</sup>*Petersburg Nuclear Physics Institute,  
Gatchina 188300, St. Petersburg, Russia*

<sup>2</sup>*Institut für Theoretische Physik II,  
Ruhr-Universität, Bochum 44780, Germany*

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## Abstract

If the number of colors  $N_c$  is taken large, baryons and their excitations can be considered in a mean-field approach. We argue that the mean field in baryons breaks spontaneously the spherical and  $SU(3)$  flavor symmetries, but retains the  $SU(2)$  symmetry of simultaneous rotations in space and isospace. The one-quark and quark-hole excitations in the mean field, together with the  $SU(3)$  rotational bands about them determine the spectrum of baryon resonances, which turns out to be in satisfactory accordance with reality when one puts  $N_c = 3$ . A by-product of this scheme is a confirmation of the light pentaquark  $\Theta^+$  baryon  $uudd\bar{s}$  as a typical Gamov–Teller resonance long known in nuclear physics. An extension of the same large- $N_c$  logic to charmed (and bottom) baryons leads to a prediction of a *anti-decapenta* ( $\overline{15}$ )-plet of charmed pentaquarks, two of which,  $\mathcal{B}_c^{++} = cuud\bar{s}$  and  $\mathcal{B}_c^+ = cudd\bar{s}$ , may be light and stable with respect to strong decays, and should be looked for [1].

Keywords: mean field, baryon resonances, exotics, charmed baryons, bottom baryons

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## I. RELATIVISTIC MEAN FIELD

It has been argued 30 years ago by Witten [2] that if the number of colors  $N_c$  is large, the  $N_c$  quarks of a baryon can be viewed as moving in a mean field. It is helpful to understand how baryons look like in the large- $N_c$  limit, before  $1/N_c$  corrections are considered.

At the microscopic level quarks experience only color interactions, however large  $N_c$  do not suppress gluon fluctuations: the mean field can be only ‘colorless’. An example how originally color interactions are Fierz-transformed into interactions of quarks with mesonic fields are provided by the instanton liquid model [3].

We shall thus assume that quarks in the large- $N_c$  baryon obey the Dirac equation in a background mesonic field since there are no reasons to expect quarks to be non-relativistic, especially in excited baryons. In a most general case the background field couples to quarks through all five Fermi variants. If the background field is stationary in time, it leads to the eigenvalue equation for the  $u, d, s$  quarks in the background field:

$$H\psi = E\psi,$$

$$H = \gamma^0 \left( -i\partial_i \gamma^i + S(\mathbf{x}) + P(\mathbf{x})i\gamma^5 + V_\mu(\mathbf{x})\gamma^\mu + A_\mu(\mathbf{x})\gamma^\mu\gamma^5 + T_{\mu\nu}(\mathbf{x})\frac{i}{2}[\gamma^\mu\gamma^\nu] \right), \quad (1)$$

where  $S, P, V, A, T$  are the mean fields that are matrices in flavor. In fact, the one-particle Dirac Hamiltonian (1) is generally nonlocal, however that does not destroy symmetries in which we are primarily interested. We include dynamically-generated quarks masses into the scalar term  $S$ .

The key issue is the symmetry of the mean field. From the large- $N_c$  point of view, the current strange quark mass is very small,  $m_s = \mathcal{O}(1/N_c^2)$  [4], therefore a good starting point is exact  $SU(3)$  flavor symmetry. A natural assumption, then, would be that the mean field is flavor-symmetric, and spherically symmetric. This assumption, however, leads to too many “missing resonances” in the spectrum. In addition, we know that baryons are strongly coupled to pseudoscalar mesons ( $g_{\pi NN} \approx 13$ ). It means that there is a large pseudoscalar field inside baryons; at large  $N_c$  it is a classical mean field. There is no way of writing down the pseudoscalar field that would be compatible with the  $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$  symmetry. The minimal extension of spherical symmetry is to write the “hedgehog” *Ansatz* “marrying”

the isotopic and space axes:

$$\pi^a(\mathbf{x}) = \begin{cases} n^a F(r), & n^a = \frac{x^a}{r}, \quad a = 1, 2, 3, \\ 0, & a = 4, 5, 6, 7, 8. \end{cases} \quad (2)$$

This *Ansatz* breaks the  $SU(3)_{\text{flav}}$  symmetry. Moreover, it breaks the symmetry under independent space  $SO(3)_{\text{space}}$  and isospin  $SU(2)_{\text{iso}}$  rotations, and only a simultaneous rotation in both spaces remains a symmetry, since a rotation in the isospin space labeled by  $a$ , can be compensated by the rotation of the space axes. Therefore, the *Ansatz* (2) breaks spontaneously the original  $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$  symmetry down to the  $SU(2)_{\text{iso+space}}$  symmetry. It is analogous to the spontaneous breaking of spherical symmetry by the ellipsoid form of many nuclei.

## II. QUARKS IN THE ‘HEDGEHOG’ MEAN FIELD

We shall call the  $SU(2)_{\text{iso+space}}$  symmetry of the mean field the “hedgehog symmetry”. What mesonic fields  $S, P, V, A, T$  in Eq. (1) are compatible with this symmetry? Since  $SU(3)$  symmetry is broken, all fields can be divided into three categories:

### I. Isovector fields acting on $u, d$ quarks

$$\text{pseudoscalar : } P^a(\mathbf{x}) = n^a P_0(r), \quad (3)$$

$$\text{vector : } V_i^a(\mathbf{x}) = \epsilon_{aik} n_k P_1(r),$$

$$\text{axial : } A_i^a(\mathbf{x}) = \delta_{ai} P_2(r) + n_a n_i P_3(r),$$

$$\text{tensor : } T_{ij}^a(\mathbf{x}) = \epsilon_{aij} P_4(r) + \epsilon_{bij} n_a n_b P_5(r).$$

### II. Isoscalar fields acting on $u, d$ quarks

$$\text{scalar : } S(\mathbf{x}) = Q_0(r), \quad (4)$$

$$\text{vector : } V_0(\mathbf{x}) = Q_1(r),$$

$$\text{tensor : } T_{0i}(\mathbf{x}) = n_i Q_2(r).$$

### III. Isoscalar fields acting on $s$ quarks

$$\text{scalar : } S(\mathbf{x}) = R_0(r), \quad (5)$$

$$\text{vector : } V_0(\mathbf{x}) = R_1(r),$$

$$\text{tensor : } T_{0i}(\mathbf{x}) = n_i R_2(r).$$

All the rest fields and components are zero as they do not satisfy the  $SU(2)$  symmetry and/or the needed discrete  $C, P, T$  symmetries. The 12 ‘profile’ functions  $P_{0,1,2,3,4,5}$ ,  $Q_{0,1,2}$  and  $R_{0,1,2}$  should be eventually found self-consistently from the minimization of the mass of the ground-state baryon. However, even if we do not know those profiles, there are important consequences of this *Ansatz* for the baryon spectrum.

Given the *Ansatz*, the Hamiltonian (1) actually splits into two: one for  $s$  quarks and the other for  $u, d$  quarks. The former commutes with the angular momentum of  $s$  quarks,  $\mathbf{J} = \mathbf{L} + \mathbf{S}$ , and with the inversion of spatial axes, hence all energy levels are characterized by half-integer  $J^P$  and are  $(2J + 1)$ -fold degenerate. The latter commutes only with the ‘grand spin’  $\mathbf{K} = \mathbf{T} + \mathbf{J}$  and with inversion, hence the  $u, d$  quark levels have definite integer  $K^P$  and are  $(2K + 1)$ -fold degenerate. The energy levels for  $u, d$  quarks on the one hand and for  $s$  quarks on the other are completely different, even in the chiral limit  $m_s \rightarrow 0$ .

All energy levels, both positive and negative, are probably discrete owing to confinement. Indeed, a continuous spectrum would correspond to a situation when quarks are free at large distances from the center, which contradicts confinement. [One can model confinement by forcing the effective quark masses to grow at infinity, *e.g.*  $Q_0(\mathbf{x}) \sim R_0(\mathbf{x}) \sim \sigma r$ .]

According to the Dirac theory, all *negative*-energy levels, both for  $s$  and  $u, d$  quarks, have to be fully occupied, corresponding to the vacuum. It means that there must be exactly  $N_c$  quarks antisymmetric in color occupying all (degenerate) levels with  $J_3$  from  $-J$  to  $J$ , or  $K_3$  from  $-K$  to  $K$ ; they form closed shells that do not carry quantum numbers. Filling in the lowest level with  $E > 0$  by  $N_c$  quarks makes a baryon [4, 5], see Fig. 1.

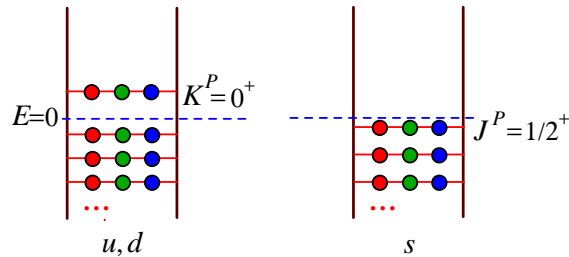


FIG. 1: Filling  $u, d, s$  shells for the ground-state baryons:  $(8, 1/2^+)$ ,  $(10, 3/2^+)$ .

The mass of a baryon is the aggregate energy of all filled states, and being a functional of the mesonic field it is proportional to  $N_c$  since all quark levels are degenerate in color. Therefore quantum fluctuations of mesonic field in baryons are suppressed as  $1/N_c$  so that

the mean field is indeed justified.

Quantum numbers of the lightest baryons are determined from the quantization of the rotations of the mean field, leading to specific  $SU(3)$  multiplets that reduce at  $N_c=3$  to the octet with spin  $\frac{1}{2}$  and the decuplet with spin  $\frac{3}{2}$ , see *e.g.* [6]. Witten's quantization condition  $Y' = \frac{N_c}{3}$  [7] follows trivially from the fact that there are  $N_c$   $u, d$  valence quarks each with the hypercharge  $\frac{1}{3}$  [8]. Therefore, the ground state shown in Fig. 1 entails in fact 56 rotational states. The splitting between the centers of the multiplets  $(\mathbf{8}, \frac{1}{2}^+)$  and  $(\mathbf{10}, \frac{3}{2}^+)$  is  $\mathcal{O}(1/N_c)$ , and the splittings inside multiplets can be determined as a perturbation in  $m_s$  [8].

### III. EXCITED STATES IN THE MEAN FIELD

The lowest baryon resonance beyond the rotational excitations of the ground state is the singlet  $\Lambda(1405, \frac{1}{2}^-)$ . Apparently, it can be obtained only as an excitation of the  $s$  quark, and its quantum numbers must be  $J^P = \frac{1}{2}^-$  [4], see transition 1 in Fig. 2.

The existence of an  $\frac{1}{2}^-$  level for  $s$  quarks automatically implies that there is a particle-hole excitation of this level by an  $s$  quark from the  $\frac{1}{2}^+$  level. We identify this transition 2 with  $N(1535, \frac{1}{2}^-)$  [4]. It is predominantly a pentaquark state  $u(d)uds\bar{s}$  (at  $N_c=3$ ). This explains its large branching ratio in the  $\eta N$  decay [9], a long-time mystery. We also see that, since the highest filled level for  $s$  quarks is lower than the highest filled level for  $u, d$  quarks,  $N(1535, \frac{1}{2}^-)$  must be *heavier* than  $\Lambda(1405, \frac{1}{2}^-)$ : the opposite prediction of the non-relativistic quark model has been always of some concern. Subtracting  $1535 - 1405 = 130$ , we find that the  $\frac{1}{2}^+$   $s$ -quark level is approximately 130 MeV lower in energy than the valence  $0^+$  level for  $u, d$  quarks.

The low-lying Roper resonance  $N(1440, \frac{1}{2}^+)$  requires an excited one-particle  $u, d$  state with  $K^P = 0^+$  [4], see transition 3. Just as the ground state nucleon, it is part of the excited  $(\mathbf{8}', \frac{1}{2}^+)$  and  $(\mathbf{10}', \frac{3}{2}^+)$  split as  $1/N_c$ . Such identification of the Roper resonance solves another problem of the non-relativistic model where  $N(1440, \frac{1}{2}^+)$  must be heavier than  $N(1535, \frac{1}{2}^-)$ . In our approach they are unrelated.

Given that there is an excited  $0^+$  level for  $u, d$  quarks, one can put there an  $s$  quark as well, taking it from the  $s$ -quark  $\frac{1}{2}^+$  shell, see transition 4. It is a particle-hole excitation with the valence  $u, d$  level left untouched, its quantum numbers being  $S = +1$ ,  $T = 0$ ,  $J^P = \frac{1}{2}^+$ . At  $N_c = 3$  it is a pentaquark state  $uudd\bar{s}$ , precisely the exotic  $\Theta^+$  baryon predicted in

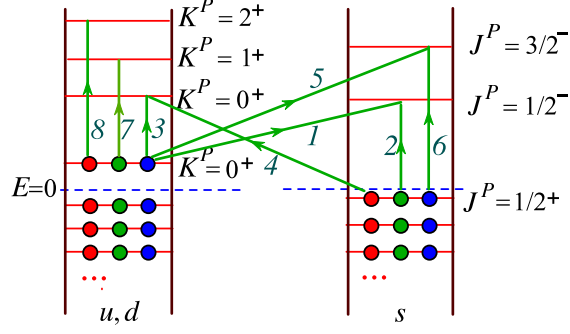


FIG. 2: All baryon resonances below 2 GeV follow from this scheme of one-quark levels. The transitions shown by arrows correspond to: 1:  $\Lambda(1405, 1/2^-)$ , 2:  $N(1535, 1/2^-)$ , 3:  $N(1440, 1/2^+)$ , 4:  $\Theta^+(1530, 1/2^+)$ , 5:  $\Lambda(1520, 3/2^-)$ , 6:  $N(1650, 1/2^-?)$ , 7:  $N(1710, 3/2^+)$ , 8:  $N(1680, 5/2^+)$ . Other resonances belong to  $SU(3)$  multiplets obtained as rotational excitations of these one-particle and particle-hole excitations.

Ref. [10] from other considerations. The quantization of its rotations produces the antidecuplet  $(\overline{\mathbf{10}}, \frac{1}{2}^+)$ . In our original prediction the  $\mathcal{O}(1)$  gap between  $\Theta^+$  and the nucleon was due to the rotational energy only, whereas here the main  $\mathcal{O}(1)$  part of that gap is due to the one-particle levels, while the rotational energy is  $\mathcal{O}(1/N_c)$ . Methodologically, it is more satisfactory.

In nuclear physics, excitations generated by the axial current  $j_{\mu 5}^\pm$ , when a neutron from the last occupied shell is sent to an unoccupied proton level or *v.v.* are known as Gamov–Teller transitions [12]. Thus our interpretation of the  $\Theta^+$  is that it is a Gamov–Teller-type resonance long known in nuclear physics.

An unambiguous feature of our picture is that **the exotic pentaquark is a consequence of the three well-known resonances and must be light**. Indeed, the  $\Theta^+$  mass can be estimated from the sum rule [4]:  $m_\Theta \approx 1440 + 1535 - 1405 \approx 1570$  MeV, however there are  $\mathcal{O}(m_s)$  corrections to this equation.

To account for higher baryon resonances one has to assume that there are higher one-particle excitations, both in the  $u, d$ - and  $s$ -quark sectors, shown in Fig. 2. It is easy to obtain that order of levels under mild assumptions about the profile functions (3)–(5).

#### IV. BARYON RESONANCES FROM ROTATIONAL BANDS

The original  $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$  symmetry is restored when flavor and space rotations are accounted for. Each transition in Fig. 2 generally entails “rotational bands” of  $SU(3)$  multiplets with definite spin and parity. The short recipe of getting them is: Find the hypercharge  $Y'$  from the number of  $u, d, s$  quarks involved; only those multiplets are allowed that contain this  $Y'$ . Take an allowed multiplet and read off the isospin(s)  $T'$  of particles at this value of  $Y'$ . The allowed spin of the multiplet obeys the angular momentum addition law:  $\mathbf{J} = \mathbf{T}' + \mathbf{J}_1 + \mathbf{J}_2 + \mathbf{K}_1 + \mathbf{K}_2$  where  $J_{1,2}$  and  $K_{1,2}$  are the initial and final momenta of the  $s$  and  $u, d$  shells involved in the transition, respectively. The mass of the center of a multiplet does not depend on  $\mathbf{J}$  but only on  $\mathbf{T}'$  according to the relation [11]

$$\mathcal{M} = \mathcal{M}_0 + \frac{C_2(p, q) - T'(T' + 1) - \frac{3}{4}Y'^2}{2I_2} + \frac{T'(T' + 1)}{2I_1} \quad (6)$$

where  $C_2(p, q) = \frac{1}{3}(p^2 + q^2 + pq) + p + q$  is the quadratic Casimir eigenvalue of the multiplet,  $I_{1,2} = \mathcal{O}(N_c)$  are moments of inertia. After the rotational band for a given transition is constructed, one has to check if the rotational energy of a particular multiplet is  $\mathcal{O}(1/N_c)$  and not  $\mathcal{O}(1)$ , and if it is compatible with Fermi statistics at  $N_c=3$ : some *a priori* possible multiplets drop out. One gets a satisfactory description of all baryon resonances up to about 2 GeV, to be published separately.

#### V. CHARMED AND BOTTOM BARYONS

If one of the  $u, d$  quarks in a light baryon is replaced by a heavy  $b$  or  $c$  quark, there are still  $N_c - 1$   $u, d$  quarks left. At large  $N_c$ , they form *the same* mean field as in light baryons, with the same sequence of Dirac levels (up to  $1/N_c$  corrections). The heavy quark contributes to the mean  $SU(3)$ -symmetric field but it is a  $1/N_c$  correction, too.

The filling of Dirac levels for the ground-state  $c$  (or  $b$ ) baryon is shown in Fig. 3: there is a hole in the  $0^+$  shell for  $u, d$  quarks. Quantizing rotations of this state leads to the following  $SU(3)$  multiplets:  $(\bar{\mathbf{3}}, 1/2^+)$ ,  $(\mathbf{6}, 1/2^+)$  and  $(\mathbf{6}, 3/2^+)$ . The last two are degenerate whereas the first is split from the rest by  $\mathcal{O}(1/N_c)$ . The splitting inside multiplets is  $\mathcal{O}(m_s N_c)$ .

There are good candidates for those ground-state multiplets:  $\Lambda_c(2287)$  and  $\Xi_c(2468)$  for  $(\bar{\mathbf{3}}, 1/2^+)$ ;  $\Sigma_c(2455)$ ,  $\Xi_c(2576)$  and  $\Omega_c(2698)$  for  $(\mathbf{6}, 1/2^+)$ ; finally  $\Sigma_c(2520)$ ,  $\Xi_c(2645)$

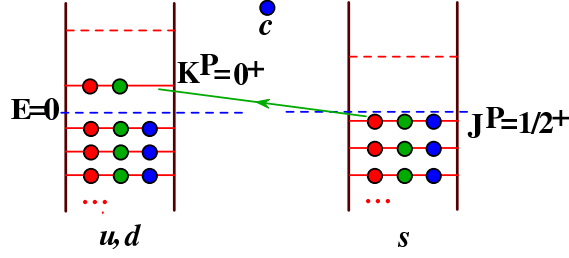


FIG. 3: Filling  $u, d, s$  shells for the ground-state charmed baryons,  $(\bar{\mathbf{3}}, 1/2^+)$ ,  $(\mathbf{6}, 1/2^+)$  and  $(\mathbf{6}, 3/2^+)$ . The arrow shows the Gamov–Teller excitation leading to charmed pentaquarks forming  $(\bar{\mathbf{15}}, 1/2^+)$ .

and  $\Omega_c(2770)$  presumably form  $(\mathbf{6}, 3/2^+)$ . There are  $\bar{\mathbf{3}}$ ’s and  $\mathbf{6}$ ’s with parity minus arising from exciting the  $1/2^-$   $s$ -quark level. The lightest are the degenerate singlets, presumably  $\Lambda_c(2595, 1/2^-)$  and  $\Lambda_c(2625, 3/2^-?)$ .

Our new observation is that there is a Gamov–Teller-type transition when axial current annihilates a strange quark in the  $\frac{1}{2}^+$  shell, and creates an  $u$  or  $d$  quark in the  $0^+$  shell, like in the case of the  $\Theta^+$ . In heavy baryons it is even more simple as there is a hole in the  $u, d$   $0^+$  valence shell from the start. Filling in this hole means making charmed pentaquarks which we name “beta baryons”,  $\mathcal{B}_c^+ = cudd\bar{s}$  and  $\mathcal{B}_c^{++} = cuud\bar{s}$ . Quantizing rotations tells us that these pentaquarks are members of the *anti-decapenta-plet*  $(\bar{\mathbf{15}}, 1/2^+)$ , Fig. 4. In fact, there must be two additional (nearly degenerate) multiplets, one with spin  $1/2^+$  and the other with spin  $3/2^+$ .

Charmed pentaquarks have been considered by Wu and Ma in another approach [13]; however, they get far larger masses and in addition pentaquarks with  $\bar{c}$  quarks appear almost degenerate with those made of  $c$  quarks. In our picture the lightest  $\bar{c}$  pentaquarks  $\Theta_c$  probably arise from putting the fourth ( $s$ ) quark at the  $\frac{1}{2}^-$  level; they form a quadruplet, have parity minus, and are much heavier.

Since we know the separation between the  $1/2^+$  level for  $s$  quarks and the  $0^+$  level for  $u, d$  quarks from fitting the light baryon resonances, and assuming that it does not change for heavy baryons (as it would be at  $N_c \rightarrow \infty$ ), we estimate the mass of the  $\mathcal{B}_c^{+,+}$  pentaquarks at about 2420 MeV! The corresponding bottom pentaquarks are about  $m(\Lambda_b) + 130 \text{ MeV} = 5750 \text{ MeV}$ . Such light charmed and bottom pentaquarks have no strong decays. Their weak decays, for example  $\mathcal{B}_c^+ \rightarrow p\phi \rightarrow pK^+K^-$ , have clear signatures especially in a vertex



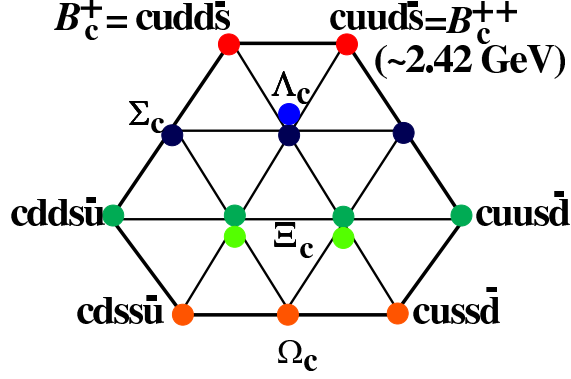


FIG. 4: Decapenta-plet of charmed pentaquarks.

detector, and should be looked for at LHC, Fermilab and B-factories. A cautionary remark, though, is that the production rate is expected to be quite low.

A detailed elaboration of the ideas presented here will be published elsewhere.

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